Exam P Formula & Review Sheet
(updated 06/28/2023)

GENERAL PROBABILITY

Conditional Probability of Event B Given Event A: $P[B|A] = \dfrac{P[B \cap A]}{P[A]}$

Probability of Event B Partitioned by Events A and A':

$$P[B] = P[B \cap A] + P[B \cap A'] = P[B|A] \cdot P[A] + P[B|A'] \cdot P[A']$$

Bayes' Rule: $$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{P[B|A] \cdot P[A]}{P[B|A] \cdot P[A] + P[B|A'] \cdot P[A']}$$

Binomial Coefficient - Number of Subsets of Size k From a Collection of n Objects: $\dbinom{n}{k} = \dfrac{n!}{k! \cdot (n-k)!}$

DeMorgan's Laws: $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$.

Inclusion-Exclusion Equations: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[B \cap C] - P[A \cap C] + P[A \cap B \cap C]$$

Central Limit Theorem: Suppose that X is a random variable with mean μ and standard deviation σ and suppose that $X_1, X_2, \cdots, X_n$ are n independent random variables with the same distribution as X.

Let $Y_n = X_1 + X_2 + \cdots + X_n$.

Then $E[Y_n] = n\mu$ and $Var[Y_n] = n\sigma^2$, and as n increases, the distribution of Y_n approaches a normal distribution $N(n\mu, n\sigma^2)$.

UNIVARIATE RANDOM VARIABLES

Distribution Function & Survival Function of Random Variable X:

$$F_X(t) = P[X \le t] \qquad S_X(t) = 1 - F_X(t) = P[X > t]$$

Expected Value of Random Variable X:

$$E[X] = \sum_{\text{all } x_i} x_i \cdot p_X(x_i) \text{ (discrete)} \qquad E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx \text{ (continuous)}$$

Moment of Random Variable X: If $n \ge 1$ is an integer, the n-th moment of X is $E[X^n]$.

If the mean of X is μ, then the n-th central moment of X is $E[(X-\mu)^n]$.

Coefficient of Variation of X: $\dfrac{\sigma_X}{\mu_X}$ or $\dfrac{\sigma_X}{E[X]}$

Median of the Distribution of Random Variable X: Smallest m for which $F_X(m) = .5$

Mode of the Distribution of Random Variable X: The point c at which $p_X(c)$ (discrete) or $f_X(c)$ (continuous) is maximized.

Discrete Uniform Distribution on the Integers $1, 2, \ldots, N$:

$p_X(k) = \frac{1}{N}$ for $k = 1, 2, \ldots N$ $\qquad E[X] = \frac{N+1}{2}$ $\qquad Var[X] = \frac{N^2-1}{12}$

Binomial Distribution for n Trials and Success Probability p:

$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$ $\qquad E[X] = np$ $\qquad Var[X] = np(1-p)$

Geometric Distribution on the Integers $0, 1, 2, \ldots$ and Parameter p:

$p_X(k) = (1-p)^k \cdot p$ $\qquad E[X] = \frac{1-p}{p}$ $\qquad Var[X] = \frac{1-p}{p^2}$

Poisson Distribution with Mean λ:

$p_X(k) = \frac{e^{-\lambda}\lambda^k}{k!}$ for $k = 0, 1, 2, \ldots$ $\qquad E[X] = \lambda$ $\qquad Var[X] = \lambda$

Negative Binomial Distribution with Parameters p and integer $\gamma \geq 1$:

$p_X(x) = \binom{\gamma + x - 1}{x} p^\gamma (1-p)^x$ for $x = 0, 1, 2, 3, \cdots$ $\qquad E[X] = \frac{\gamma(1-p)}{p}$ $\qquad Var[X] = \frac{\gamma(1-p)}{p^2}$

Hypergeometric Distribution:

$p_X(x) = \frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}}$ $\qquad E[X] = \frac{nK}{M}$ $\qquad Var[X] = \frac{nK(M-K)(M-n)}{M^2(M-1)}$

for $\max\{0, n-(M-K)\} \leq x \leq \min\{n, K\}$

Continuous Uniform Distribution on the Interval $[a, b]$:

$f_X(x) = \frac{1}{b-a}$ for $a \leq x \leq b$ $\qquad E[X] = \frac{a+b}{2}$ $\qquad Var[X] = \frac{(b-a)^2}{12}$

Standard Normal Distribution $N(0, 1)$ (Mean 0, Variance 1):

$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ for $-\infty < x < \infty$ $\qquad F_X(x) = \Phi(x)$

Normal Distribution $N(\mu, \sigma)$ (Mean μ, Variance σ^2):

$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$ for $-\infty < x < \infty$ $\qquad F_X(x) = \Phi(\frac{x-\mu}{\sigma})$

Exponential Distribution with Mean $\theta > 0$:

$f_X(x) = \frac{1}{\theta} e^{-x/\theta}$ for $x > 0$, and $f_X(x) = 0$ otherwise $\qquad E[X] = \theta$ $\qquad Var[X] = \theta^2$

Gamma Distribution with Parameters $\alpha > 0$ and $\theta > 0$:

$f_X(x) = \frac{x^{\alpha-1}e^{-x/\theta}}{\theta^\alpha \Gamma(\alpha)}$ for $x > 0$, and $f_X(x) = 0$ otherwise $\qquad E[X] = \alpha\theta$ $\qquad Var[X] = \alpha\theta^2$

Lognormal Distribution with parameters μ and σ: If $\ln(Y) \sim N(\mu, \sigma^2)$ then we have

$$f_Y = \frac{1}{\sigma y\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln(y)-\mu}{\sigma}\right)^2\right), \qquad E[Y] = e^{\mu+\frac{\sigma^2}{2}} \qquad Var[Y] = E[Y]^2\left(e^{\sigma^2}-1\right).$$

Beta Distribution with parameters $\alpha > 0$ and $\beta > 0$:

$$f_X(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, \qquad E[X] = \frac{\alpha}{\alpha+\beta}, \qquad Var[X] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}.$$

for $0 < x < 1$ and 0 otherwise.

Insurance deductible: The amount paid by the insurer, $Y = \begin{cases} 0 & \text{if } X \le d \\ X-d & \text{if } X > d \end{cases} = Max(X-d, 0).$

Policy limit: The amount paid by the insurer, Y, for a claim $X = \begin{cases} X & \text{if } X \le u, \\ u & \text{if } X > u. \end{cases}$

Coinsurance: The amount paid by the insurer, $Y = \begin{cases} X & \text{if } X \le d, \\ a(X-d) & \text{if } d < X \le u, \\ a(u-d) & \text{if } X > u. \end{cases}$

Cost per loss with deductible: $F_{Y^L}(x) = \Pr(X \le x+d) = F_X(x+d)$

Cost per payment with deductible: $F_{Y^P}(x) = \dfrac{F_X(x+d) - F_X(d)}{1-F_X(d)}$

Expected Cost per loss with deductible: $E[Y^L] = \int_d^\infty (1-F_X(x))\ dx$

MULTIVARIATE RANDOM VARIABLES

Marginal Distribution of X From Joint Distribution of X and Y: $p_X(x) = \sum_{\text{all } y} p(x,y)$ (discrete)

Independence of Random Variables X and Y: $f(x,y) = f_X(x) \cdot f_Y(y)$

Conditional Distribution of X Given Y: $f_{X|Y}(x|y) = \dfrac{f(x,y)}{f_Y(y)}$

Covariance and Correlation Between X and Y: $Cov[X,Y] = E[XY] - E[X] \cdot E[Y]$

$$\rho_{XY} = \frac{Cov[X,Y]}{\sqrt{Var[X] \cdot Var[Y]}}$$

Variance of the Sum of X and Y: $Var[X+Y] = Var[X] + Var[Y] + 2Cov[X,Y]$