# A. REVIEW OF PROBABILITY

## A1. BASIC PROBABILITY

**Cumulative Distribution Function:** $F(x) = \Pr(X \leq x)$

**Survival Function:** $S(x) = 1 - F(x) = \Pr(X > x)$

**Probability Density Function:** $f(x) = \frac{d}{dx}F(x) = -\frac{d}{dx}S(x)$

**Hazard Rate Function:** $\lambda(x) = \frac{f(x)}{S(x)} = -\frac{d}{dx}\log S(x)$ $\rightarrow$ $S(x) = e^{-\int_{-\infty}^{x}\lambda(t)dt}$

**Cumulative Hazard Function:** $\Lambda(x) = \int_{-\infty}^{x}\lambda(t)\,dt$ $\rightarrow$ $S(x) = e^{-\Lambda(x)}$

**Transformation:** $Y = g(X)$ $\rightarrow$ $f_Y(y) = f_X\left(g^{-1}(y)\right)\left|\frac{dg^{-1}(y)}{dy}\right|$

## A2. EXPECTATION & VARIANCE

**Expectation (Discrete $X$):** $E[X] = \sum x \Pr(X = x)$ $\rightarrow$ $E[g(X)] = \sum g(x)\Pr(X = x)$

**Expectation (Continuous $X$):** $E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$ $\rightarrow$ $E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\,dx$

**Variance:** $Var(X) = E\left[X^2\right] - E[X]^2$

**Standard Deviation:** $SD(X) = \sqrt{Var(X)}$

**Covariance:** $Cov(X,Y) = E[XY] - E[X]E[Y]$

**Correlation Coefficient:** $Corr(X,Y) = \dfrac{Cov(X,Y)}{SD(X)\ SD(Y)}$

**Linear Combination:** $W = aX + bY$
$\rightarrow$ $E[W] = aE[X] + bE[Y]$
$\rightarrow$ $Var(W) = a^2 Var(X) + b^2 Var(Y) + 2ab\,Cov(X,Y)$

**Sum of iid $X$:** $S = X_1 + \cdots + X_n$
$\rightarrow$ $E[S] = nE[X]$
$\rightarrow$ $Var(S) = nVar(X)$

## A3. CONDITIONAL PROBABILITY

**Conditional Probability:** $\Pr(X=x|Y=y) = \frac{\Pr(X=x,\ Y=y)}{\Pr(Y=y)} = \frac{\Pr(X=x)\Pr(Y=y|X=x)}{\Pr(Y=y)}$

**Conditional Density Function:** $f(x|y) = \frac{f(x,y)}{f(y)} = \frac{f(x)f(y|x)}{f(y)}$

**Conditional expectation (Discrete):** $E[X|Y=y] = \sum x \Pr(X=x|Y=y)$

**Conditional expectation (Continuous):** $E[X|Y=y] = \int_{-\infty}^{\infty} x f(x|y)dx$

**Law of Total Probability:** $\Pr(X \le x) = E[\Pr(X \le x|Y)]$

**Law of Total Expectation:** $E[X] = E[E[X|Y]]$

**Law of Total Variance:** $Var(X) = E[Var(X|Y)] + Var(E[X|Y])$

# B. SEVERITY, FREQUENCY & AGGREGATE MODELS

## B1. SEVERITY DISTRIBUTIONS

| Distribution | Probability density function | Formulas worth memorizing | | |
|---|---|---|---|---|
| Uniform | $f(x) = \frac{1}{b-a},\ a \le x \le b$ | $F(x) = \frac{x-a}{b-a}$ | $E[X] = \frac{a+b}{2}$ | $Var(X) = \frac{(b-a)^2}{12}$ |
| Exponential | $f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}},\ x>0$ | $F(x) = 1-e^{-\frac{x}{\theta}}$ | $E[X] = \theta$ | $Var(X) = \theta^2$ |
| Weibull | $f(x) = \frac{\tau}{x}\left(\frac{x}{\theta}\right)^{\tau} e^{-\left(\frac{x}{\theta}\right)^{\tau}},\ x>0$ | $F(x) = 1-e^{-\left(\frac{x}{\theta}\right)^{\tau}}$ | | |
| Gamma | $f(x) = \frac{\left(\frac{1}{\theta}\right)^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1}e^{-\frac{x}{\theta}},\ x>0$ | $F(x) = \Pr(X^* \ge \alpha)$<br>$X^*$ is Poisson with $\lambda = \frac{x}{\theta}$<br>If $\alpha$ is an integer. | $E[X] = \alpha\theta$ | $Var(X) = \alpha\theta^2$ |
| Beta | $f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$,<br>$0<x<1$ | | $E[X] = \frac{a}{a+b}$<br>If $a$ and $b$ are integers. | $E[X^2] = \frac{a(a+1)}{(a+b)(a+b+1)}$<br>If $a$ and $b$ are integers. |
| Pareto | $f(x) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}},\ x>0$ | $F(x) = 1-\left(\frac{\theta}{x+\theta}\right)^{\alpha}$ | $E[X] = \frac{\theta}{\alpha-1}$<br>If $\alpha>1$ is an integer. | |
| Single P. Pareto | $f(x) = \frac{\alpha\theta^{\alpha}}{x^{\alpha+1}},\ x>\theta$ | $F(x) = 1-\left(\frac{\theta}{x}\right)^{\alpha}$ | $E[X] = \frac{\alpha\theta}{\alpha-1}$ | |
| Lognormal | $f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\log x-\mu)^2}{2\sigma^2}},\ x>0$ | $F(x) = N\left(\frac{\log x-\mu}{\sigma}\right)$ | $E[X] = e^{\mu+\frac{\sigma^2}{2}}$ | $E[X^2] = e^{2\mu+2\sigma^2}$ |

## B2. FREQUENCY DISTRIBUTIONS

| Distribution | Probability mass function | Formulas worth memorizing | |
|---|---|---|---|
| Poisson | $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots, \infty$ | $E[N] = \lambda$ | $Var(N) = \lambda$ |
| Binomial | $P(x) = \binom{m}{x} q^x (1-q)^{m-x}, x = 0, 1, 2, \ldots, m$ | $E[N] = mq$ | $Var(N) = mq(1-q)$ |
| Geometric | $P(x) = \left(\frac{1}{1+\beta}\right)\left(\frac{\beta}{1+\beta}\right)^x, x = 0, 1, 2, \ldots, \infty$ | $E[N] = \beta$ | $Var(N) = \beta(1+\beta)$ |
| Negative Binomial | $P(x) = \binom{x+r-1}{x}\left(\frac{1}{1+\beta}\right)^r\left(\frac{\beta}{1+\beta}\right)^x, x = 0, 1, 2, \ldots, \infty$ | $E[N] = r\beta$ | $Var(N) = r\beta(1+\beta)$ |

**Zero-truncated distributions.:** $p_x^T = \frac{p_x}{1-p_0}$

**Zero-modified distributions.:** $p_x^M = \left(1 - p_0^M\right)\left(\frac{p_x}{1-p_0}\right)$

## B3. AGGREGATE MODELS

**Individual risk model:** $S = X_1 + X_2 + \cdots + X_n \rightarrow E[S] = nE[X]$

$\rightarrow Var(S) = nVar(X)$

**Collective risk model:** $S = X_1 + X_2 + \cdots + X_N \rightarrow E[S] = E[N]E[X]$

$\rightarrow Var(S) = E[N]Var(X) + Var(N)E[X]^2$

**Normal Distribution:** $X_i \overset{iid}{\sim} N\left(\mu,\ \sigma^2\right) \rightarrow S^L = X_1 + X_2 + \cdots + X_n \sim N\left(n\mu,\ n\sigma^2\right)$

**Exponential Distribution:** $X_i \overset{iid}{\sim} Exp(\theta) \rightarrow S^L = X_1 + X_2 + \cdots + X_n \sim Gamma(\alpha = n,\ \theta)$

**Normal approximation:** $S \dot\sim N\left(\mu = E[S], \sigma^2 = Var(S)\right) \rightarrow Pr(S \leq k) \approx N\left(\frac{k-\mu}{\sigma}\right)$

## B4. MEASURES OF RISKS

**Coherent risk measures:**

**1. Translation Invariance:** $\rho(X + c) = \rho(X) + c$

**2. Positive Homogeneity:** $\rho(cX) = c\rho(X)$

**3. Subadditivity:** $\rho(X + Y) \leq \rho(X) + \rho(Y)$

**4. Monotonicity:** $\rho(X) \leq \rho(Y)$ If $\Pr(X \leq Y) = 1$

**Value-at-Risk:** $\text{VaR}_\alpha(X)$ Where $\Pr(X \leq \text{VaR}_\alpha(X)) = \alpha$.

**Tail-Value-at-Risk:** $\text{TVAR}_\alpha(X) = E[X|X > \text{VaR}_\alpha(X)] = \text{VaR}_\alpha(X) + \frac{E[X] - E[X \wedge \text{VaR}_\alpha(X)]}{1-\alpha}$

**Equilibrium Distribution:** $f_e(x) = \frac{S(x)}{E[X]},\ X > 0 \rightarrow E[X_e] = \frac{E[X^2]}{2E[X]}$

# C. COVERAGE MODIFICATIONS

## C1. PAYMENT PER LOSS

| Policy | Payment per loss | Expected payment per loss |
|---|---|---|
| With ordinary deductible $d$ | $Y^L = \begin{cases} 0, & X < d \\ X-d, & X \geq d \end{cases}$ | $E\left[Y^L\right] = E[X] - E[X \wedge d]$ |
| With franchise deductible $d^*$ | $Y^L = \begin{cases} 0, & X \leq d^* \\ X, & X > d^* \end{cases}$ | $E\left[Y^L\right] = E\left[X \mid X > d^*\right]$ |
| With maximum covered loss $u$ | $Y^L = \begin{cases} X, & X \leq u \\ u, & X > u \end{cases}$ | $E\left[Y^L\right] = E[X \wedge u]$ |
| With $d$ and $u$ | $Y^L = \begin{cases} 0, & X \leq d \\ X-d, & d < X \leq u \\ u-d, & X > u \end{cases}$ | $E\left[Y^L\right] = E[X \wedge u] - E[X \wedge d]$ |
| With $d,\ u$ and coinsurance factor $\alpha$ | $Y^L = \begin{cases} 0, & X \leq d \\ \alpha(X-d), & d < X \leq u \\ \alpha(u-d), & X > u \end{cases}$ | $E\left[Y^L\right] = \alpha(E[X \wedge u] - E[X \wedge d])$ |
| With $d, u, \alpha$ and inflation rate $r$ | $Y^L = \begin{cases} 0, & X \leq \frac{d}{1+r} \\ \alpha(1+r)\left(X - \frac{d}{1+r}\right), & \frac{d}{1+r} < X \leq \frac{u}{1+r} \\ \alpha(1+r)\left(\frac{u}{1+r} - \frac{d}{1+r}\right), & X > \frac{u}{1+r} \end{cases}$ | $E\left[Y^L\right] = \alpha(1+r)\left(E\left[X \wedge \frac{u}{1+r}\right] - E\left[X \wedge \frac{d}{1+r}\right]\right)$ |

**Loss elimination ratio:** $\text{LER} = 1 - \frac{E\left[Y^L\right]}{E[X]}$

## C2. PAYMENT PER PAYMENT

**Payment per Payment** $Y^P = Y^L | X > d \quad \rightarrow \quad E\left[Y^P\right] = \frac{E\left[Y^L\right]}{\Pr(X > d)}$

| Policy | Loss | Payment per payment |
|---|---|---|
| With ordinary deductible $d$ | $X \sim Unif(0, b)$ | $Y^P \sim Unif(0, b-d)$ |
| | $X \sim Exp(\theta)$ | $Y^P \sim Exp(\theta)$ |
| | $X \sim Pareto(\alpha, \theta)$ | $Y^P \sim Pareto(\alpha, \theta + d)$ |

### C3. REINSURANCE

| Proportional reinsurance | Insurer pays | Reinsurer pays |
|---|---|---|
| With quota share $\alpha$ | $Y=(1-\alpha)X$ | $Y=\alpha X$ |
| With retention $u$ and surplus share $\alpha$ | $Y=\begin{cases}X, & X\le u\\ u+(1-\alpha)(X-u), & X>u\end{cases}$ | $Y=\begin{cases}0, & X\le u\\ \alpha(X-u), & X>u\end{cases}$ |

| Excess of loss reinsurance | Insurer pays | Reinsurer pays |
|---|---|---|
| Covers losses above $d$ | $Y=\begin{cases}X, & X\le d\\ d, & X>d\end{cases}$ | $Y=\begin{cases}0, & X\le d\\ X-d, & X>d\end{cases}$ |
| Covers losses above $d$ but below $u$ | $Y=\begin{cases}X, & X\le d\\ d, & d\le X<u\\ X+d-u, & X>u\end{cases}$ | $Y=\begin{cases}0, & X\le d\\ X-d, & d\le X<u\\ u-d, & X>u\end{cases}$ |

# D. MAXIMUM LIKELIHOOD ESTIMATION

### D1. MLE WITH COMPLETE DATA

For distributions that belong to the exponential family:

1. Determine $L(\theta)$.
2. Apply natural logarithm, obtain $l(\theta)=\log L(\theta)$.
3. Take the first derivative with respect to the parameter, obtain $l'(\theta)$.
4. Set $l'(\theta)=0$, obtain $\hat{\theta}$, which is the MLE.

| Distribution | Likelihood Function | Maximum likelihood estimate(s) | |
|---|---|---|---|
| Exponential | $L(\theta)=f(x_1)\dots f(x_n)$ | $\hat{\theta}=\bar{x}$ | |
| Normal | | $\hat{\mu}=\bar{x}$ | $\hat{\sigma}^2=\frac{1}{n}\sum(x_i-\hat{\mu})^2$ |
| Lognormal | | $\hat{\mu}=\frac{1}{n}\sum\log x_i$ | $\hat{\sigma}^2=\frac{1}{n}\sum(\log x_i-\hat{\mu})^2$ |
| Uniform | | $\widehat{b}=\max(x_1,\dots,x_n)$ | |
| Binomial | $L(\theta)=p(x_1)\dots p(x_n)$ | $\hat{q}=\frac{\bar{x}}{m}$ | |
| Poisson | | $\hat{\lambda}=\bar{x}$ | |

## D2. MLE WITH INCOMPLETE DATA

| | Likelihood function | Note |
|---|---|---|
| Grouped data | $L = (F(c_1) - F(c_0))^{m_1} \ldots (F(c_n) - F(c_{n-1}))^{m_n}$ | Where $c_0 < c_1 < \cdots < c_n$ are interval boundaries. |
| Left-truncated data | $L = \dfrac{f(x_1)}{S(d)} \ldots \dfrac{f(x_n)}{S(d)}$ | Losses below $d$ are not reported. |
| Right-censored data | $L = f(x_1) \ldots f(x_n) S(u)^m$ | Losses are capped at $u$. |
| Left-truncated & Right-censored data | $L = \dfrac{f(x_1)}{S(d)} \ldots \dfrac{f(x_n)}{S(d)} \left(\dfrac{S(u)}{S(d)}\right)^m$ | Losses below $d$ are not reported. Losses are capped at $u$. |

# E. CLASSICAL CREDIBILITY

## E1. FULL CREDIBILITY

| We want…to be within $k$ of the mean $p$ of the time. | Range parameter, $k$ Note: $z = N^{-1}\left(\dfrac{1+p}{2}\right)$ | Total number of exposures needed, $e_F$ | Total number of claims needed, $n_F$ |
|---|---|---|---|
| Average number of claims | $k = \dfrac{z\sqrt{\dfrac{\text{Var}(N)}{e_F}}}{E[N]}$ | $e_F = \left(\dfrac{z}{k}\right)^2 \times \dfrac{\text{Var}(N)}{E[N]^2}$ | $n_F = \left(\dfrac{z}{k}\right)^2 \times \dfrac{\text{Var}(N)}{E[N]^2} \times E[N]$ |
| Total number of claims | $k = \dfrac{z\sqrt{e_F \text{Var}(N)}}{e_F E[N]}$ | | |
| Average claim size | $k = \dfrac{z\sqrt{\dfrac{\text{Var}(X)}{n_F}}}{E[X]}$ | $e_F = \left(\dfrac{z}{k}\right)^2 \times \dfrac{\text{Var}(X)}{E[X]^2} \times \dfrac{1}{E[N]}$ | $n_F = \left(\dfrac{z}{k}\right)^2 \times \dfrac{\text{Var}(X)}{E[X]^2}$ |
| Average aggregate claims | $k = \dfrac{z\sqrt{\dfrac{\text{Var}(S)}{e_F}}}{E[S]}$ | $e_F = \left(\dfrac{z}{k}\right)^2 \times \dfrac{\text{Var}(S)}{E[S]^2}$ | $n_F = \left(\dfrac{z}{k}\right)^2 \times \dfrac{\text{Var}(S)}{E[S]^2} \times E[N]$ |
| Total aggregate claims | $k = \dfrac{z\sqrt{e_F \text{Var}(S)}}{e_F E[S]}$ | | |

## E2. POISSON FREQUENCY

| We want…to be within $k$ of the mean $p$ of the time. | Frequency | Total number of exposures needed, $e_F$ | Total number of claims needed, $n_F$ |
|---|---|---|---|
| Average number of claims | $N \sim Poisson(\lambda)$ | $e_F = \left(\dfrac{z}{k}\right)^2 \times \dfrac{1}{\lambda}$ | $n_F = \left(\dfrac{z}{k}\right)^2$ |
| Total number of claims | | | |
| Average claim size | | $e_F = \left(\dfrac{z}{k}\right)^2 \times \dfrac{\text{Var}(X)}{E[X]^2} \times \dfrac{1}{\lambda}$ | $n_F = \left(\dfrac{z}{k}\right)^2 \times \dfrac{\text{Var}(X)}{E[X]^2}$ |
| Average aggregate claims | | $e_F = \left(\dfrac{z}{k}\right)^2 \times \left(1 + \dfrac{\text{Var}(X)}{E[X]^2}\right) \times \dfrac{1}{\lambda}$ | $n_F = \left(\dfrac{z}{k}\right)^2 \times \left(1 + \dfrac{\text{Var}(X)}{E[X]^2}\right)$ |
| Total aggregate claims | | | |

## E3. PARTIAL CREDIBILITY

**Credibility factor:** $Z = \sqrt{\dfrac{\text{Number of exposures available}}{\text{Number of exposures needed for full credibility}}}$

or $Z = \sqrt{\dfrac{\text{Number of claims available}}{\text{Number of claims needed for full credibility}}}$

**Credibility premium:** $P = Z \times \text{Observation} + (1 - Z) \times \text{Manual Rate}$

# F. RATEMAKING & LOSS RESERVING

## F1. RATEMAKING DATA

| | |
|---|---|
| **Exposure** | Written exposure<br>Earned exposure<br>Unearned exposure |
| **Premium** | Written premium<br>Earned premium<br>Unearned premium |
| **Claim** | Reported claims<br>Unreported claims |
| **Loss** | Reported loss = Paid loss + Case reserve<br>IBNR reserve = Incurred but not reported reserve<br>IBNER reserve = Incurred but not enough reported reserve<br>Ultimate loss = Reported loss + IBNR reserve + IBNER reserve |
| **Expenses** | ALAE = Allocated loss adjustment expenses<br>ULAE = Unallocated loss adjustment expenses |

**Basic formulas:**

$$\text{Frequency} = \frac{\text{Number of Claims}}{\text{Number of Exposures}}$$

$$\text{Severity} = \frac{\text{Losses}}{\text{Number of Claims}}$$

**Pure Premium/Loss Cost** $= \dfrac{\text{Losses}}{\text{Number of Exposures}} = \text{Frequency} \times \text{Severity}$

**Loss Ratio** $= \dfrac{\text{Losses}}{\text{Premium}}$

**Expense Ratio** $= \dfrac{\text{Expenses}}{\text{Premium}}$

## F2. AGGREGATION METHODS

| Aggregation methods | Premium/Exposure | Loss |
|---|---|---|
| Calendar year | Transaction date | Transaction date |
| Calendar-accident year | Transaction date | Accident date |
| Policy year | Effective date | Effective date |

## F3. LOSS RESERVING METHODS

| | |
|---|---|
| Expected loss ratio method | 1. Ultimate losses = Earned Premium × Expected Loss Ratio<br>2. Loss Reserve = Ultimate Losses − Paid Losses |
| Chain-ladder method | 1. Prepare a run-off triangle for paid losses.<br>2. Calculate age-to-age factors using average factor method or mean factor method.<br>3. Calculate age-to-ultimate factor $f_{ULT}$, which is the product of age-to-age factors.<br>4. Ultimate Losses = Paid Losses × $f_{ULT}$<br>5. Loss reserve = Ultimate Losses − Paid Losses |
| Bornhuetter-Ferguson method | 1. Prepare a run-off triangle for paid losses.<br>2. Calculate age-to-age factors using average factor method or mean factor method.<br>3. Calculate age-to-ultimate factor $f_{ULT}$, which is the product of age-to-age factors.<br>4. Ultimate Losses = Paid Losses + Earned Premium × Expected Loss Ratio × $\left(1 - \frac{1}{f_{ULT}}\right)$<br>5. Loss reserve = Ultimate Losses − Paid Losses |

## F4. PRICING FORMULA

**General formula:** Premium = Losses + Loss adjustment expenses + Fixed Expenses + Variable Expenses + Profit

**Premium:** $P = L + LAE + F + (V+Q)P \quad \rightarrow \quad P = \dfrac{L + LAE + F}{1 - V - Q}$

**Permissible loss ratio:** $R = 1 - V - Q \quad \rightarrow \quad P = \dfrac{L + LAE + F}{R}$

**Adjustments to data:** $\text{Premium at current rates} = \text{Earned premium} \times \dfrac{\text{Current rate level}}{\text{Historical average rate level}}$

Ultimate losses = Reported losses × Development factor

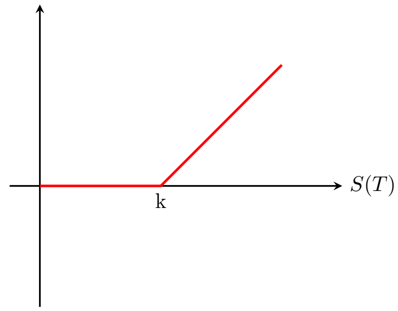
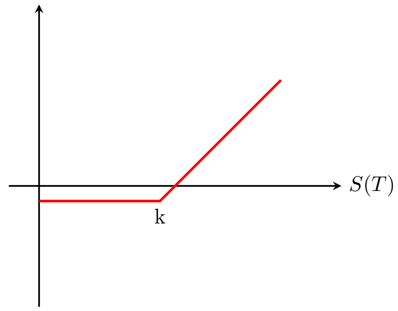
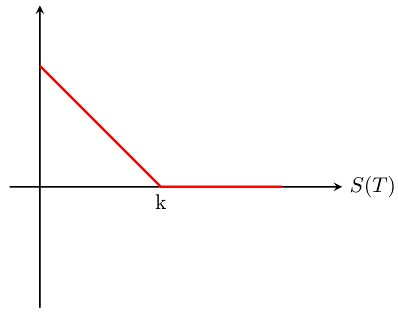
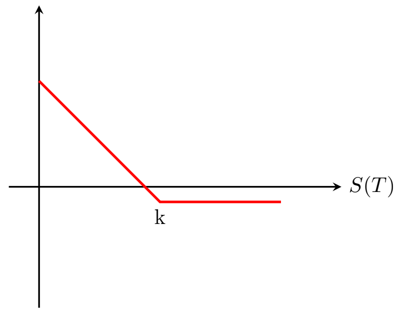
Trended losses = Reported losses × Trend factor

## F5. RATEMAKING METHODS

| | |
|---|---|
| **Loss cost method** | $\text{Projected loss cost including LAE} = \dfrac{\text{Trended and ultimate losses and LAE}}{\text{Number of earned exposures}}$ <br> $\text{Indicated rate} = \dfrac{\text{Projected loss cost+Fixed expenses per exposure}}{\text{Permissible loss ratio}}$ <br> $\text{Indicated rate change} = \dfrac{\text{Indicated rate}}{\text{Current rate}} - 1$ |
| **Loss ratio method** | $\text{Projected loss and LAE ratio} = \dfrac{\text{Trended and ultimate losses and LAE}}{\text{Earned premiums at current rate level}}$ <br> $\text{Indicated rate change} = \dfrac{\text{Projected loss and LAE ratio} + \text{Fixed expense ratio}}{\text{Permissible loss ratio}} - 1$ <br> $\text{Indicated rate} = \dfrac{\text{Earned premiums at current rate level}}{\text{Number of earned exposures}} \times (1 + \text{Indicated rate change })$ |

# G. Option Pricing

## G1. PUT AND CALL OPTIONS

| Financial derivative | Payoff | Profit |
|---|---|---|
| **Call option** | $C(T) = \max(0, S(T) - K)$   | $\text{Profit} = C(T) - C(0)e^{rT}$   |
| **Put option** | $P(T) = \max(0, K - S(T))$   | $\text{Profit} = P(T) - P(0)e^{rT}$   |

**Put-call parity:** $C(0) - P(0) = S(0) - Ke^{-rT}$

## G2. BINOMIAL OPTION PRICING MODEL

**Stock price at time $h$:** $S_u = S(0) \times u$ or $S_d = S(0) \times d$

**Option payoff at time $h$:** $V_u$ or $V_d$

**Replicating portfolio:** $\Delta = \frac{V_u - V_d}{S_u - S_d}$ & $B = e^{-rh}\left(\frac{uV_u - dV_u}{u-d}\right)$

**Option price:** $V(0) = \Delta S(0) + B$

**Risk neutral probability:** $p^* = \frac{e^{rh} - d}{u - d}$

**Option price:** $V(0) = e^{-rh}\left(p^* V_u + (1-p^*) V_d\right)$

## G3. BLACK-SCHOLES-MERTON MODEL

**Stock price at time $T$:** $S(T) = S(0)e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}Z}$ where $Z \sim N(0,1)$

**Call price:** $C(0) = S(0)N(d_1) - Ke^{-rT}N(d_2)$

**Call delta:** $\Delta_C = N(d_1)$

**Put price:** $P(0) = Ke^{-rT}N(-d_2) - SN(-d_1)$

**Put delta:** $\Delta_p = -N(-d_1)$

**Where:** $d_1 = \frac{\log\frac{S(0)}{K} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ & $d_2 = d_1 - \sigma\sqrt{T}$